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Galerkin Method and Axisymmetric Vibrations of Polar-Orthotropic Circular Plates

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Nomenclature

A_i, A_j = undetermined constants
a = radius of plate
$D_r, D_\theta, D_{r\theta}$ $= E_r(h^3/12), E_\theta(h^3/12), E_{r\theta}(h^3/12)$
$E_r, E_\theta, E_{r\theta}$ = elastic constants of the plate material
h = thickness
k $= D_\theta / D_r$
r, θ = polar coordinates
W = transverse deflection amplitude
W_a = approximate transverse deflection amplitude
ρ = mass density of the plate material
ω = circular frequency
Ω_{oo} = fundamental frequency coefficient $[=\sqrt{(\rho h/D_r)}\,\omega_{oo}a^2]$

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Introduction

TRANSVERSE, axisymmetric vibrations of circular plates of polar orthotropy have been analyzed by several authors.[1-3] However, several inconsistencies appear in these studies, as has been first pointed out by Leissa[4] and discussed in greater detail by Prathap and Varadan.[5] These authors point out that the inconsistencies are generated because certain boundary conditions are ignored or treated incorrectly in Refs. 1-3 when using the Galerkin method. Prathap and Varadan[5] show that, in the case of a clamped orthotropic plate, convergence is achieved if the displacement amplitude is approximated using a polynomial approach, i.e.,

$$W_a = \sum_{i=0}^{I} A_i \left[1 - \left(\frac{r}{a}\right)^2\right]^{2+i} \tag{1}$$

and generating the frequency equation by means of the Lagrangian formulation.

Conversely, it is shown in Ref. 6 that polynomial coordinate functions yield excellent accuracy in the case of elastically restrained plates of circular orthotropy subject to an in-plane state of hydrostatic stress when the Ritz formulation is used.

It is the object of this study to show the existence of a type of coordinate, polynomial functions that, in spite of the fact that they do not comply with the "internal" condition[5]:

$$(1-k^2)\frac{d^2 W}{dr^2}(O) = 0, \qquad k = \frac{D_\theta}{D_r} \tag{2}$$

converge in a very satisfactory fashion when the Galerkin formulation is invoked.

Discussion of the Methodology and Numerical Results

If one considers the response of the orthotropic, circular plate subjected to a uniformly distributed transverse load, one has the following functional relations[7] for the displacement function $w(r)$:

$$w(r) = A + Cr^{1+k} + qr^4/8(9-k^2)D_r \tag{3}$$

where A and C are determined using the governing boundary conditions.

One notices immediately the existence of the term r^{1+k}. Accordingly, it seems reasonable to try coordinate functions where the parameter k is contained in a similar fashion.

It was found convenient to use the approximation:

$$W(r) \simeq W_a(r) = \sum_{j=0}^{J} A_j \left[\alpha_j \left(\frac{r}{a}\right)^{3+k} + \beta_j \left(\frac{r}{a}\right)^{1+k} + 1\right]\left(\frac{r}{a}\right)^{4j} \tag{4}$$

Table 1 Comparison of fundamental frequency coefficients $\sqrt{\frac{\rho h}{D_r}}\,\omega_{oo}a^2$

$a/\phi D_r$	$k=\frac{D_\theta}{D_r}$	μ_θ	Ref. 6, Ritz method	Ref. 4	Present study
	0.25	0.22	2.994	2.500	2.497 (1 term) 2.497 (2 terms) 2.497 (3 terms)
	0.5	0.4	3.781	3.629	3.628 3.627 3.627
	0.5	0.3	3.607	3.452	3.453 3.452 3.452
	0.75	0.7	4.795	4.765	4.772 4.768 4.767
	1	0.75	5.516	5.518	5.534 5.523 5.521
0	1.25	1	6.478	6.472	6.505 6.480 6.474
	1.25	0.5	5.937	5.934	5.955 5.936 5.932
	1.5	0.75	6.951	6.906	6.972 6.934 6.925
	1.5	0.5	6.678	6.646	6.693 6.659 6.651
	1.75	0.35	7.249	7.188	7.267 7.216 7.204
	0.75	—	9.508	9.457	9.506 9.471 9.463
	1	—	10.217	10.215	10.327 10.262 10.242
∞	1.4	—	11.498	11.453	11.685 11.545 11.508
	1.5	—	11.831	11.760	12.032 11.869 11.825

Obviously, when $k=1$ (isotropic case) the "base function" is:

$$\alpha_j(r/a)^4+\beta_j(r/a)^2+1 \qquad (5)$$

which coincides with the coordinate function previously used by Laura and co-workers when dealing with isotropic plates.[6]

Numerical Results and Discussion

Table 1 presents a comparison of fundamental frequency coefficients obtained using different approaches in the case of simply supported and clamped plates and for several combinations of k and μ_θ. One observes that the Galerkin method yields excellent accuracy even when a single-term approximation is used. Apparently, the convergence attained using the Galerkin method and the coordinate functions defined in Eq. (4) is quite satisfactory if one considers the comparison of results shown also in Table 2 ($k<1$) and Table 3 ($k>1$).

For $N(a^2/D_r)=0$ and $k=0.50$ and 0.75, the Galerkin method yields frequency values that are in excellent agreement

Table 2 Comparison of values of the fundamental frequency coefficient $\sqrt{\frac{\rho h}{D_r}}\,\omega_{oo}a^2$ as a function of k, $\frac{a}{\phi D_r}$, and $N\frac{a^2}{D_r}$ $(k<1)$; $\mu_\theta=0.30$

		$N(a^2/D_r)=0$					$N(a^2/D_r)=10$				
		Ref. 6		Present study			Ref. 6		Present study		
	$a/\phi D_r$	Ritz	Finite elements	1 term	2 terms	3 terms	Ritz	Finite elements	1 term	2 terms	3 terms
$k=0.50$	0	3.607	3.452	3.453	3.452	3.452	8.417	9.065	8.368	8.368	8.367
	10^{-1}	3.781	3.627	3.628	3.627	3.627	8.493	9.143	8.443	8.443	8.442
	1	4.901	4.748	4.751	4.749	4.749	9.061	9.728	9.007	9.007	9.006
	10	7.603	7.397	7.410	7.400	7.398	10.990	11.765	10.918	10.912	10.910
	10^5	8.935	8.685	8.705	8.690	8.687	12.265	13.158	12.164	12.158	12.156
$k=0.75$	0	4.227	4.199	4.203	4.200	4.200	8.702	8.997	8.695	8.691	8.691
	10^{-1}	4.384	4.366	4.360	4.357	4.356	8.779	9.074	8.771	8.767	8.767
	1	5.431	5.401	5.410	5.404	5.403	9.351	9.653	9.338	9.337	9.337
	10	8.118	8.073	8.111	8.086	8.081	11.321	11.668	11.297	11.297	11.297
	10^5	9.508	9.450	9.506	9.470	9.463	12.650	13.048	12.618	12.618	12.618

Table 3 Comparison of values of the fundamental frequency coefficient $\sqrt{\frac{\rho h}{D_r}}\,\omega_{\infty}a^2$ as a function of k, $\frac{a}{\phi D_r}$, and $N\frac{a^2}{D_r}$ $(k>1)$; $\mu_\theta=0.30$

		$N(a^2/D_r)=0$					$N(a^2/D_r)=10$				
		Ref. 6		Present study			Ref. 6		Present study		
	$a/\phi D_r$	Ritz	Finite elements	1 term	2 terms	3 terms	Ritz	Finite elements	1 term	2 terms	3 terms
$k=1.25$	0	5.679	5.666	5.693	5.676	5.673	9.509	9.271	9.543	9.518	9.512
	10^{-1}	5.812	5.798	5.827	5.810	5.806	9.585	9.347	9.620	9.595	9.589
	1	6.748	6.733	6.780	6.751	6.745	10.167	9.924	10.202	10.177	10.171
	10	9.454	9.434	9.569	9.489	9.470	12.247	11.969	12.293	12.265	12.256
	10^5	11.004	10.981	11.169	11.061	11.033	13.723	13.405	13.784	13.750	13.737
$k=1.50$	0	6.435	6.396	6.444	6.414	6.408	9.993	9.567	10.045	10.000	9.989
	10^{-1}	6.559	6.520	6.572	6.540	6.533	10.070	9.643	10.122	10.077	10.065
	1	7.456	7.411	7.490	7.442	7.431	10.656	10.220	10.712	10.664	10.651
	10	10.187	10.120	10.326	10.203	10.172	12.794	12.296	12.876	12.814	12.793
	10^5	11.831	11.748	12.032	11.868	11.825	14.353	13.784	14.461	14.386	14.358

with the extremely accurate results determined using a finite element approach developed by Pardoen.[6] The results show a more marked difference for $k=1.25$ and 1.50.

It is important to point out that for $N(a^2/D_r)=10$, $k=0.50$ and 0.75, the Galerkin method yields results that are considerably lower than the finite element values. In view of the fact that the Galerkin method yields upper bounds, they are presumably more accurate.

However, for $k=1.25$ and 1.50, $N(a^2/D_r)=10$, the situation reverses since the values obtained by means of the Galerkin method are considerably higher.

It is observed that, in general, a one-term approximation yields sufficient accuracy, at least from the point of view of many engineering applications. The entire algorithm can be easily implemented on a desk computer, however. The approach can be easily extended in the case of other complications: variable thickness, concentrated masses, etc.

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